Varying c cosmology and Planck value constraints

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Abstract

It has been suggested that by increasing the speed of light during the early universe various cosmological problems of standard big bang cosmology can be overcome, without requiring an inflationary phase. However, we find that as the Planck length and Planck time are then made correspondingly smaller, and together with the need that the universe should not re-enter a Planck epoch, the higher c models have very limited ability to resolve such problems. For a constantly decreasing c the universe will quickly becomes quantum gravitationally dominated as time increases: the opposite to standard cosmology where quantum behaviour is only ascribed to early times.

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1 Introduction

It is well known that the standard big bang model (SBB) model has a number of worrying puzzles, particulary the so-called horizon and flatness problems, that generally relate to the fixing of arbitrary constants. But, as emphasized by Zeldovich [1] perhaps the most fundamental and serious being the fact that the energy density is much larger than the Planck value $\sim 10^{93} gcm^{-3}$ if the present universe is run back to when the universe was Planck size. Equivalently to account for this discrepancy we require that the size of the universe be already much larger than the Planck length (l_{pl}) for time $\sim$ Planck time (t_{pl}). This mismatch of scales is generally referred to as the Planck problem of SBB cosmology. This is true of any matter source that obeys the strong-energy condition, so including radiation or dust sources.

1.1 The Planck problem

Consider just a radiation source with a FRW metric, the Friedmann equation is,

$$H^2 + \frac{c^2 k}{a^2} = \frac{A}{a^4} \quad (A = \text{constant}) \tag{1}$$

where we set Newton's constant $8G/3 = 1$, but keep c explicit but constant throughout section (1). The solution of this equation, ignoring the curvature k, is simply $a = A^{1/4}|t|^{1/2}$, with $t = 0$ being the initial singularity. We can now highlight the Planck problem that occurs with a big bang model with such a matter source. If such a model is to account for our present universe then the constant A has to be extremely large $\sim 10^{120}$. Consider a universe created with Planck radius ($\sim 10^{-33}$ cm) and Planck density ($\sim 10^{93}\text{gcm}^{-3}$). If such a universe expands to its present size greater than $\sim 10^{28}$cm then the density would be of order [1]

$$\rho \sim 10^{93} \left(10^{28}/10^{-33}\right)^{-4} \simeq 10^{-151}\text{gcm}^{-3} \tag{2}$$

This should be compared to the present energy density $\sim 10^{-30}\text{gcm}^{-3}$. Even if the radiation energy density was immediately converted into dust the resulting energy density would still be $\sim 10^{-90}\text{gcm}^{-3}$. To account for this discrepancy we require the constant A to be so large $\sim 10^{120}$ that the energy density is now vastly greater (a factor A) than the Planck value for when the universe is $\sim$ Planck size . This also forces the size of the universe $a \sim A^{1/4} t^{1/2}$ to be much bigger than Planck size for time $\sim$ Planck time (t_p).

This Planck problem is, as said, in many ways the most fundamental problem we first need to solve with a cosmological model. Otherwise we will fail to understand the enormous size and matter content of our actual universe. This problem is present in flat $k = 0$ universe since for a natural value of $A \sim 1$ the size of a radiation dominated universe with scale factor $a \sim t^{1/2}$, with today's lifetime $10^{60} t_p$ is only $\sim 10^{-33}\text{cm} * 10^{30} \sim 10^{-3}\text{cm}!$. But now including the $A^{1/4} \sim 10^{30}$ factor gives a more correct $\sim 10^{27}$ cm value.

1.2 The horizon problem

The horizon problem occurs because the the particle horizon size, defined as

$$r = c \int_0^t \frac{dt}{a(t)} \tag{3}$$

is finite, see eg.[2-4]. The horizon proper distance R is this quantity r multiplied by the scale factor i.e $R = a{*}r$. For any strong-energy satisfying matter source this quantity R grows linearly with time. But in SSB cosmology the rate of change of the scale factor, given by $a \sim t^p$ and $1/3 < p < 1$, grows increasingly rapidly as $t \to 0$. The horizon cannot keep pace with the scale factor 'velocity' $\dot{a} \sim 1/t^{1-p}$. But note that this is only impossible for times below unity $0 < t < 1$. If the horizon problem was solved, by some process, at the Planck time $t_{pl} = t = 1$ it would remain permanently solved during the ensuing evolution. For the inflationary value $p > 1$ the horizon problem is simply avoided. One can further understand this by noting that the usual space-like singularity of the FRW universe becomes a null singularity when $p > 1$- see eg.[5].

1.3 The flatness problem

Consider a perfect-fluid equation of state: $p = (\gamma - 1)\rho$. The Friedmann equation is again

$$H^2 + \frac{c^2 k}{a^2} = \rho \tag{4}$$

There is also the continuity equation

$$\dot{\rho} + 3H\gamma\rho = 0 \quad \Rightarrow \quad \rho = \frac{A}{a^{3\gamma}} \tag{5}$$

With A the previously introduced constant. The density parameter Ω, defined as $\Omega = \rho/H^2$ can be written as, see eg.[2],

$$\Omega = \frac{A}{A - c^2 k a^{3\gamma - 2}} \tag{6}$$

If the strong energy condition is satisfied i.e. $\gamma > 2/3$ then as the scale factor $a \to 0$, Ω is set initially to 1. For increasing time t the value of Ω diverges as [2]

$$|\Omega - 1| \propto t^{2-4/3\gamma} \tag{7}$$

We can estimate the value of Ω at the Planck time and assuming $\gamma = 4/3$ throughout the evolution of the universe. The age of the universe is $\sim 10^{60} t_{pl}$. Then using expression (6) we can relate Ω at different times as

$$\frac{(\Omega - 1)_{now}}{(\Omega - 1)_{then}} \approx 10^{60} \tag{8}$$

If we assume that today $\Omega \approx 1$ then at the Planck time we require $(\Omega - 1) < 10^{-60}$, i.e. $\Omega \sim 1 \pm 10^{-60}$. The flatness problem can be considered as being effectively solved by having an exceedingly big value of $\dot{a}$ at the Planck time. This sets the density parameter Ω, where

$$\Omega = 1 + \frac{c^2 k}{\dot{a}^2} \quad , \tag{9}$$

extremely close to unity so that even today at time $\sim 10^{60} t_p$ it has still not departed significantly from unity. Again the large value of the constant $A \sim 10^{120}$ can achieve this since for radiation $\dot{a}^2 = A^{1/2} t^{-1}$.

So far we have not included any inflationary early stage However, with inflation the Planck problem is helped by having a huge expansion while the energy density remains roughly constant. This obviates the need for arbitrary constants that usually set parameters, particularly $\dot{a}$, vastly post-Planckian where quantum gravity is utterly dominant. With inflation, the constant 'A' is automatically forced large without requiring an unnaturally large initial value -see eg.[3]. We should add that the flatness problem might not actually be a problem at all in SBB cosmology and is rather a question of how one 'picks' the arbitrary constant 'A'. If the equations had a different form or transformed to different variables then a large 'A' might be quite natural. This is indeed the case when one considers an invariant canonical measure for the classical solutions [6], or works with a DeWitt superspace approach [7] or even with Bayesian reasoning [8]. Likewise, as emphasized by Padmanabhan the horizon problem is also essentially a quantum gravitational problem as changing the behaviour of the scale factor, just while $t < t_{pl}$, can generally resolve the problem [4]. For these reason we regard the Planck problem as

being the fundamental puzzle of non-inflationary (SBB) cosmology and to which alternative models, here the variable speed of light, must help resolve.

2.0 Variable c cosmology

It has been suggested [9,10,11] that by changing the speed of light c during the early universe the various puzzles can be solved. One is effectively resetting the constant A above to be unity in new units. Consider the Planck values

$$m_{pl} = \sqrt{\frac{\hbar c}{G}} \quad : \quad l_{pl} = \sqrt{\frac{\hbar G}{c^3}} \quad : \quad t_{pl} = \sqrt{\frac{\hbar G}{c^5}} \tag{10}$$

Note that although the Planck mass increases with c, both the Planck size and Planck time decrease more rapidly with c: this will be shown to be the source of a fatal flaw with such alterations of c.

First consider the Planck density $\sim m_{pl}/l_{pl}^3$ this scales as c^5 , so an increase in c of order 10^{20} would appear to possibly resolve the Planck density problem by increasing it by $\sim 10^{120}$ times. It is suggested [9,10] a bigger increase of 10^{30} in c is actually needed to resolve the flatness problem but this actual amount will not alter our arguments. This higher c value now allows one to apply the classical equations up to the now enormous Planck density of $\sim 10^{240} gcm^{-3}$. We note in passing that the hierarchy problem of why the masses of elementary particles are much less than the allowed Planck mass, would seem exacerbated with a higher m_{pl} cf.[12].

2.1 Sudden switch in c

In the first model [9,10] the speed of light was considered to undergo a phase transition and its value to suddenly fall by, say, a factor $\sim 10^{30}$. We will, as in ref.[10], represent the first region, with higher c, by a $(-)$ and the subsequent region with c taking its present value by a $(+)$, i.e. $c_+ = 10^{-30} c_-$. Now consider region $(-)$, since c is fixed the Friedman equations remain valid. Although we do not have a correct measure to apply at the Planck epoch in regime $(-)$ we will assume that initially roughly equipartition is valid and that $\Omega \simeq 1$ being given by a radiation source. This takes place now for Planck size $l_{pl}^- \sim 10^{-45} * 10^{-33} cm \sim 10^{-78} cm$ and Planck time $t_{pl}^- \sim 10^{-75} * 10^{-44}$ s $\sim 10^{-119}$s.

We cannot change the value of c until the scale factor and time are greater than the Planck values in region $(+)$ otherwise the universe is simply left stranded within the quantum gravitational epoch when we have no realistic idea of what happens. We refer to this as achieving Planck epoch escape.

But this means that the high c region has to exist for 10^{75} of its Planck units before the time becomes 10^{-44} seconds, or t_{pl}^{+} which is also the present Planck time with our value of c. During this period Ω will diverge away from unity in the usual way $|\Omega - 1| \propto t$ but it now has a longer time to diverge $\sim 10^{75}$ compared to the usual radiation big bang model with present time 10^{60} Planck units.

Consider first the case of closed ($k = 1$) universes. Keeping other constants fixed the maximum size of the closed universe a_{max} is reduced for increased c, actually $a_{max} \propto c^{-1}$ -cf. eq.(11) below. It is even more unlikely that any closed universe will survive to time $t_{pl}^{+} \sim 10^{-44}$s than a standard big bang model will survive to our present age. Things are little better for the open ($k = -1$) cases. Because the initial value of Ω is not initially highly tuned to be unity the curvature will rapidly dominate the dynamics. Once the curvature dominates the solution becomes of the Milne form and the expansion rate increases. It is now a faster $a \sim t$ rate compared to radiation $a \sim t^{1/2}$ and this contributes more dilution of the radiation matter term since $\rho \sim a^{-4}$. For example consider the case that Ω is initially $1 - O(10^{-60})$, the same required amount of fine tuning as in the usual big bang model. For the first $10^{60} t_{pl}^{-}$ the scale factor grows $10^{30} l_{pl}^{-}$. For the remaining time $10^{15} t_{pl}^{-}$ until t_{pl}^{+} the scale factor grows a further $10^{15} l_{pl}^{-}$, being driven faster in a curvature dominated phase. In total the scale factor has grown $10^{45} l_{pl}^{-}$ which is just equal to l_{pl}^{+}. When the speed of light now changes the curvature is diluted by a factor $c^2 \sim 10^{60}$ so once more the value of Ω is $1 - O(10^{-60})$ - so no actual improvement has been made to the fine-tuning of Ω. If the initial value of Ω was less fine tuned than $O(10^{-60})$ the curvature would have dominated earlier and the radiation diluted more: the value of Ω would still be roughly zero even after the speed of light had changed to its lower value. Although this Milne curvature phase was anticipated in ref.[10] their analysis had not taken into account the altered Planck units and they missed the extremely long period of time, in the Planck units of region $(-)$, that passes before a phase transition can occur. During this time the matter is being rapidly diluted, in total as in the above example, by a factor bigger than can be compensated by the later switch in c. One still needs a mechanism to produce matter with $\Omega \simeq 1$ at $t_{pl}^{+} \sim 10^{-44}$s.

Making the change in the speed of light even bigger would not help, in fact, it will make the Planck time t_{pl}^{-} even smaller and allow even more time for Ω to depart from unity. Working with a dust equation of state ($\gamma = 1$)

gives, allowing for changes in expansion rate , a similar result. It might be thought that decreasing γ would eventually allow one to succeed, but recall the density now decreases slower $\rho \propto a^{-3\gamma}$. One must wait until the initial density $\sim 10^{240} gcm^{-3}$ falls below $\sim 10^{93} gcm^{-3}$ before one can change the value of c, cf. eq.(2) above.

To conclude, the flatness problem is either worsened or just remains the same depending on the initial degree of fine tuning. We note in passing that a matter source with 'curvature' equation of state $\gamma = 2/3$ would not be diluted, and the change in c could indeed set the value of $\Omega = 1 \pm O(10^{-60})$. But this, so-called 'coasting' solution case is already known to be borderline inflationary [2].

As for the Planck problem, it has, essentially by fiat, been solved by redefining the large constant 'A' to unity but then we need to understand why c then suddenly changes later in the universe's evolution. This has to occur at a time huge in the Planck time units of this high c universe when 'quantum gravitational' effects are then not expected to dominate. Why c should change simultaneously over such scales is also unclear. Since the transition proceeds rapidly the regions are rapidly loosing causal contact and then why they should all choose the same c_+ seems a further complication. The Planck problem has just been rewritten in a new guise which is now just as arbitrary and unexplained: previously we did not understand why 'A' was large, then constants are set to make it appear natural with the value unity, but this then requires that c change by a huge factor $\sim 10^{30}$ much later in the evolution of the universe: again the ubiquitous mismatch of scales.

The horizon problem does not seem to be explained as well as by using inflation. The $(-)$ region has its own horizon problem as $t \rightarrow 0$; $\dot{a} \rightarrow \infty$: so the expansion rate can always 'outdo' c. Any model with an expansion $a \sim t^p$ with $0 < p < 1$ will suffer this divergence in $\dot{a}$ as $t \rightarrow 0$. Unlike inflation one cannot get the universe emanating from a single region that has always being in causal contact as $t \rightarrow 0$. Why the horizon problem is then 'solved' just because it exists for a time $10^{75} t_{pl}^{-}$ makes no more sense than saying the horizon problem would be presently solved in any SBB model that grew to our present universe. Some mechanism is required to explain this smoothing and also the fact that fluctuations that can grow during this $10^{75} t_{pl}^{-}$ period will need to be sufficiently erased. Arguments based on perturbation theory will hardly suffice given such large time scales for evolution cf.[9,10]. Simply claiming the Jeans mass is never reached during the high c period, so that

no structures can form, seems simply wishful thinking cf.[10]. There is also a dubious argument [9] to give a scale invariant spectrum; but this uses the $(+)$ Planck values in the $(-)$ region and anyway uses an inflationary result: the presence of Hawking radiation, for the generation of fluctuations. Also note that any cosmological constant will have ample time to become dominant before c switches unless Λ, for some reason, is already extremely small in the $(-)$ region.

To summarize the horizon problem: although superficially it appears that the horizon $\sim ct$ can be resolved by a big increase in c the natural time unit is correspondingly reduced at a faster rate $\propto c^{-5/2}$. The Planck horizon size $\sim ct$ correspondingly falls from 10^{-33}cm to 10^{-79}cm. This means that the universe has to exist for huge times to create a sufficiently large causal region, but even then no 'smoothing mechanism' is presented. If one postulates such a mechanism then why doesn't this mechanism still keep the universe smooth today as it is also 'only' at age $10^{60}t^{+}_{pl}$? Essentially this is just another way of saying that one can always re-set units so that $c = 1$ in region $(-)$ and no extra phenomena is really being introduced.

To conclude this section: little advantage has been found by invoking a sudden change in the speed of light. The flatness problem remains and the Planck problem is just transformed as to: why the speed of light should change by an enormous amount when the universe is hugely larger than its natural Planck units? Instead we next consider the alternative and more extreme contention that c is continually changing.

2.2 Gradual changing of c

It is also possible that the speed of light changes gradually instead of being a sudden jump. Although we leave aside worries that this would seem to contradict various experimental data dating back to shortly after the big bang: the c changers would contend that other variables would also change to compensate. Consider now the relevant equations [10,11,13].

$$H^2 + \frac{c^2(t)k}{a^2} = \rho \tag{11}$$

$$\dot{\rho} + 3H\left(\rho + \frac{p}{c^2(t)}\right) = \frac{2kc\dot{c}}{a^2} \tag{12}$$

The matter obeying the usual equation of state

$$p = (\gamma - 1)\rho c^2(t) \tag{13}$$

Taking the speed of light to alter with the scale factor, such that

$$c = c_o a^n \tag{14}$$

with $c_o > 0$ and n constants.

Because there are no longer two distinct regions where Planck constants can be fixed the analysis is more involved but is constrained in a similar fashion. Recall that we need to ensure that the scale factor remains larger than the Planck length, and of course that the age of the universe never be less than the current Planck time. At the same time we need to be diluting the curvature more rapidly than the fall off in matter density to ensure Ω stays near unity. Solving the above equations it is found that [13]

$$\frac{\Omega}{\Omega - 1} = Ba^{2-2n-3\gamma} + \frac{C}{2n - 2 + 3\gamma} \quad , \tag{15}$$

with A and B constants. To give flatness ($\Omega \to 1$) as $a \to \infty$ requires $n < (2 - 3\gamma)/2$ where the expansion asymptotes to the usual $a \sim t^{2/3\gamma}$ behaviour [11,13,14]. The same bound can also solve the horizon problem [11,13].

We concentrate on the Planck escape aspect which is first necessary to resolve but which will be found to then constrain whether the flatness and horizon problems can also be simultaneously solved. For an expansion $a \sim t^{2/3\gamma}$ the time goes as $t \propto a^{3\gamma/2}$. This should be contrasted with the Planck time t_{pl} which scales as $\sim c^{-5/2}$, or using the relation above $t_{pl} \propto a^{-5n/2}$. Since we require that $t > t_{pl}$ for increasing a to stay away from the quantum gravitational epoch we get a constraint on the allowed negativity of n, such that $n > -3\gamma/5$. This should now be contrasted with the required value that was obtained to resolve the flatness problem $n < (2 - 3\gamma)/2$. These two constraints are now only compatible for $0 \leq \gamma < 10/9$, so excluding the important radiation case $\gamma = 4/3$. One has extended the inflationary producing value of $0 \leq \gamma \leq 2/3$ that always resolves the flatness and horizon problems just up to 10/9. Although this now includes, unlike the sudden change in c example, the dust ($\gamma = 1$) case this seems a large price to pay for such an advantage. I also leave aside doubts that in resolving the horizon problem for $2/3 < \gamma < 10/9$, one is still taking the variables 'out of bounds' into the quantum gravitational regime.

Now it is then further argued [11,13,14] that the increasing speed of light does also have the further advantage, over inflation, of resolving the cosmological constant problem: why $\Lambda \simeq 0$. But this requires an even larger negative n such that $n < -3\gamma/2$ [11,13] which is again not compatible with the Planck epoch escape requirement for any value of γ.

Alternatively for $0 > n > -3\gamma/2$ it was claimed [11,13] that, so-called quasi-flatness could be achieved: where the ratio of matter density to cosmological constant approaches a constant. But for this the scale factor then goes as $a \propto t^{-1/n}$ [11,13]. This means $t \propto a^{-n}$ and comparing with the Planck time $t_{pl} \propto a^{-5n/2}$ gives $t/t_{pl} \propto a^{3n/2}$. So this behaviour of the scale factor would now require $n > 0$ to escape the Planck epoch which is in contradiction with the solution i.e. $n < 0$, that would give quasi-flatness. The necessity of Planck epoch escape makes it no longer possible to have a sufficiently changing speed of light to set $\Lambda \to 0$ or even one that could achieve quasi-flatness. On would need to go outside the range of validity of variable c General relativity theory in the attempt to solve the cosmological constant problem. In other words one is actually working within the unknown Planck domain in order to solve the various problems. It is known, without even changing c, that simply ignoring the Planck epoch circumvents the flatness and Planck problems: essentially because all FRW universes are equivalent modulo arbitrary constants cf.[6-8]. In an apparent response to these sorts of criticism Barrow and Magueijo [15] try to defend the constantly changing c theory. For the case of radiation they give the expression $t_{pl} \propto t^{-n}$. [1]

To solve the various problems requires $n < -1$, for the sake of argument take $n = -2$, then $t_{pl} \propto t^2$. In ref.[15] they note that as $t \to 0$, t_{pl} approaches zero more rapidly than t so that in some sense the Planck time is never approached. But I have rather emphasized the problem that occurs for $t >> 1$ when $t_{pl}/t \propto t$. So it is more 'natural' for t_{pl} to be greater than t unless arbitrary large constants are imposed to prevent this. You might try and argue that we never reach cosmic time $t > 1$ by using some large time units, but this would simply introduce another problem of why there are different time scales. [2]

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[1] I seem to obtain the relation $t_{pl} \propto t^{-5n/4}$ but this discrepancy does not alter the following arguments.

[2] As mentioned in a purely classical model there is no natural time scale or 'ruler' to judge scales by, but in practice there are natural length scales cf. [8] and such problems of scale cannot be transformed away.

In the limit $n \to -1$ we obtain the relation $t_{pl} \propto t$ which is still unsatisfactory since at present $t/t_{pl} \sim 10^{60}$ and we wish to understand this large number.

The decreasing value of c is making the Planck values rapidly grow and eventually overtake the regions of classical validity. In some sense the universe starts out in a classical domain for $t \to 0$ and then becomes quantum gravitational dominated as $t > 1$. This is the opposite of how the big bang is usually perceived in that quantum gravity is assumed necessary as $t \to 0$. The advocates have assured me that "there is no principle stating that the universe must be created in the Planck epoch", but this does not seem a virtue.

The lack of an initial quantum gravitational epoch means the theory is never superseded and the various expressions and initial conditions have to be simply taken as given without any further explanation. A quantum gravitational epoch might also have helped describe or ameliorate the still present singularity at $a \to 0$. In any case there is still a, now apparently redundant, quantum gravitationally scale in the universe that becomes increasingly important in the future (actually for when $t > 1$) as c decreases. They say that "instabilities of the big bang are converted into attractors by varying c": but it seems that along with this the quantum epoch is also switched, to occur at now future times.

To conclude, the only advantage we find with a constantly changing c is that there is a slight improvement in the resolution of the flatness and horizon problems from the usual strong energy violating or inflationary values $0 \leq \gamma \leq 2/3$ to now $0 \leq \gamma < 10/9$. This seems a high price to pay for such little improvement especially when other possible problems could further restrain how c may vary, which would decide what values of n are allowed. This advantage is further offset by then needing to explain why c, although initially tending to ∞ at the initial singularity, changes at some unexplained rate. Incidentally this ∞ being the reason why the horizon problem can be solved compared to the previous finite sudden change in c. The initial singularity is also effectively null in this case for infinite c.

We have not allowed other constants to vary, particularly Newton's constant G, and this might allow further scope to give a natural explanation of SBB cosmology. However, we remain rather skeptical. A related attempt to use Brans-Dicke gravity (so G can vary) in the, so-called, pre-big bang cosmology also suffers a related Planck problem [16]. This is because the

strong-energy condition is likewise not being violated and it is then not clear why the universe is being driven large unless arbitrary constants cf. 'A' are again picked to be huge. Switching around constants seems more an exercise in 'rearranging the deck chairs' while what seems to be of more fundamental use is actually making gravity repulsive: inflation.

This might be somewhat unfair as allowing changing 'constants' might have other more aesthetic advantages. But these advantages need to be carefully assessed and placed alongside the disadvantage of requiring somewhat ad hoc assumptions of how these 'constants' should change. To avoid simply rewriting puzzles in new ways, one needs a theory that gives a certain prediction for how these changes should occur.

Recall, that anyway one can simulate some of the possible advantages of higher c values without actually altering c or any of the usual fundamental constants: examples include allowing wormholes during the early universe [17] or simply having more extreme geometries with closed timelike curves [18].

References

1. Y.B. Zeldovich, "My Universe: selected reviews", Harwood Academic Press (1992) p.95.

2. G.F.R Ellis, "Proc. Banff Summer Research Institute on Gravitation" eds. R. Mann and P. Wesson (World Scientific: Singapore) (1991). Class. Quant. Grav. 5 (1988) p. 891.

3. A.D. Linde, "Particle Physics and Inflationary cosmology", (Harwood: Switzerland) (1990). E.W. Kolb and M.S. Turner, "The Early Universe" (Addison-Wesley:New York) (1990).

4. T. Padmanabhan, *Structure formation in the universe* Cambridge University press: Cambridge (1993) p.359.

5. J.M.M. Senovilla, Gen. Rel. Grav. 30 (1998) p.701.

6. S.W. Hawking and D.N. Page, Nucl. Phys. B 298 (1988) p.789. D.H. Coule, Class. Quant. Grav. 12 (1995) p.455.

7. H.T. Cho and R. Kantowski, Phys. Rev. D 50 (1994) p.6144.

8. G. Evrard and P. Coles, Class. Quant. Grav. 12 (1995) p.L93. D.H. Coule, Class. Quant. Grav. 13 (1996) p.2029.

9. J.W. Moffat, Int. J. of Mod. Phys. A 2 (1993) p.351.

10. A. Albrecht and J. Magueijo, Phys. Rev. D 59 p.043516.

11. J.D. Barrow, Phys. Rev. D 59 (1999) p.043515.

12. A.D. Sakharov, Sov. Phys. Doklady 12 (1968) p.1040.

13. J.D. Barrow and J. Magueijo, "Solutions to the Quasi-Flatness and Quasi-Lambda problems", preprint astro-ph/9811073.

14. J.D. Barrow and J. Magueijo, "Varying-α Theories and solutions to the cosmological problems", preprint astro-ph/9811072.

15. J.D. Barrow and J. Magueijo, " Solving the flatness and quasi-flatness problems in Brans-Dicke cosmologies with a varying light speed",preprint astro-ph/9901049.

16. D.H. Coule, Class. Quant. Grav. 15 (1998) p.2803;
Phys. Lett. B 450 (1999) p.48.

17. L.Liu, F. Zhao and Li-Xin Li, Phys. Rev. D 52 (1995) p.4752.

18. J.R. Gott and Li-Xin Li, Phys. Rev. D 58 (1998) p. 023501.